Designing Optimal Crop Revenue Insurance*

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Abstract

The optimal crop revenue insurance contract is designed from recent developments in the theory of insurance economics under incomplete markets. The message is twofold. First, when the indemnity schedule is contingent on individual price and individual yield, the optimal contract depends only on the individual gross revenue. Second, this policy is shown to fail if the indemnity function is based on aggregate price and/or aggregate yield. A closed-form solution, which could be easily implemented, is proposed. It differs from actual revenue insurance programs proposed to the U.S. farmers. When insurance and capital markets are unbiased, the first-best solution can be replicated with existing crop yield and revenue insurance policies and hedging contracts if the decision variables are not constrained.

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Introduction

This paper is a first attempt to investigate the design of an optimal crop revenue insurance contract through recent theoretical developments in the literature of insurance economics. When the indemnity schedule depends on the individual price and the individual yield, the optimal indemnity schedule turns out to be contingent only on the individual gross revenue, defined as the individual yield times the price at which the producer sells his output. However, real-world markets often are not complete because the indemnity function is based on imperfect estimators of the individual yield and/or price. For instance, under Income Protection and Crop Revenue Coverage programs, indemnity payments are based on national output price rather than individual price. The optimal revenue insurance is designed in this context of incomplete markets. Indemnity payments are contingent not only on the aggregate gross revenue, equal to the aggregate yield multiplied by the aggregate price, but also on the aggregate price and/or on the aggregate yield. A closed-form solution of the optimal revenue insurance contract, which could be easily implemented, is proposed.

The second purpose of this paper is to examine how the optimal revenue insurance policy could be replicated with crop and revenue insurance policies and with hedging contracts against price risk offered by real-world insurance and financial markets. The results are used to comment on empirical findings about the risk-reducing performance of insurance and hedging contracts recently presented in the agricultural economic literature.

Optimum Insurance Design against Individual Yield and Price Risk

In the Expected Utility framework, the producer's preferences are represented by a monotone increasing and strictly concave von Neumann-Morgenstern utility function u. His gross revenue is exposed to multiple uncertainty, such as yield risk and price risk, that affect each crop. The stochastic gross revenue is defined as a deterministic function R of n nonnegative random variables $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_i, ..., \tilde{x}_n)$ with a joint density function $f(x_1, ..., x_n)$ defined over the support $X \equiv [0, \bar{x}_1] \times ... \times [0, \bar{x}_n]$, where $\bar{x}_i > 0$ for $i = 1, ..., n$. Hence, the producer's gross revenue is $R(x)$ when $x = (x_1, ..., x_n) \in X$ is realized. The function R is assumed to increase with respect to each argument:

(1) $R_i(x) \equiv \partial R(x) / \partial x_i \geq 0$ for all $x \in X$, for $i = 1, ..., n$.

To protect against the occasional occurrence of low revenue levels, the producer may purchase an insurance contract. It is described by a couple $[I(.),P]$ where $I(x)$ is the payment transferred from the insurance company to the insured producer when x is realized, and P is the insurance premium. It should be noted that the indemnity function is assumed to depend upon individual random parameters, such as individual yields and prices at which the farmer sells his production. A feasible indemnity function must be nonnegative:

(2) $I(x) \geq 0$ for all $x \in X$

and the premium is assumed to depend only on the actuarial value of the policy:

(3) $P = c[EI(\tilde{x})]$

with $c(0) = 0$, $c'(I) \geq 1$ for all $I \geq 0$ and E is the expectation operator.

An optimal insurance contract is obtained by finding the insurance premium and the indemnity function that maximize the insured producer's expected utility function of gross revenue under the constraints above:

(4) $$\max_{I(.),P} Eu[R(\tilde{x}) + I(\tilde{x}) - P] = \int_{x\in X} u[R(x_1, x_2,..., x_n) + I(x_1, x_2,..., x_n) - P] f(x_1, x_2,..., x_n) dx_1 dx_2 ... dx_n$$

subject to conditions (2) and (3).

This problem is solved by using Kuhn-Tucker conditions.

PROPOSITION 1. *The optimal insurance contract, solution to program (4) subject to constraints (2) and (3) when P is fixed, provides full coverage below a trigger gross revenue $\hat{R} \geq 0$: $I^*(x) = \max(\hat{R} - R(x), 0)$.*

In addition, one can show that if insurance is sold at an actuarially fair price, i.e. $c'(I) = 1$ for all $I \geq 0$, $\hat{R}$ is equal to the maximum gross revenue $\overline{R} = R(\overline{x}_1, \overline{x}_2,..., \overline{x}_n)$. Therefore, the insured producer chooses to be fully insured: his gross revenue net of the indemnity and the insurance premium is equal to its expectation in all states of the world. When insurance is costly, i.e. $c'(I) > 1$ for some $I \geq 0$, $\hat{R}$ is lower than $\overline{R}$.

The superiority of revenue insurance has been highlighted when the indemnity payments are contingent on individual revenue losses. However real-world markets often are not complete in that indemnity schedule is based on an imperfect signal of the producer's final wealth. We examine in the next section the design of an optimal insurance against joint yield and price uncertainty in this context of incomplete markets.

Optimum Insurance Design against Aggregate Yield and Price Risk

The indemnity schedule is now assumed to be based on imperfect estimators of individual yield and price. These indexes can represent aggregate yield estimated in a surrounding geographic area and futures price on board of trade. To model the imperfect mechanism provided by the insurance markets, individual yield $\tilde{y}_i$ and individual price $\tilde{p}_i$ are written as a linear function of the yield index $\tilde{y}$ and price index $\tilde{p}$, respectively:

(5) $\tilde{p}_i = \boldsymbol{a}_1 + \boldsymbol{b}_1 \tilde{p} + \tilde{\boldsymbol{e}}_1$

(6) $\tilde{y}_i = \boldsymbol{a}_2 + \boldsymbol{b}_2 \tilde{y} + \tilde{\boldsymbol{e}}_2$

where $p \in [0, p_{\max}]$, $y \in [0, y_{\max}]$, $\tilde{\boldsymbol{e}}_1$ and $\tilde{\boldsymbol{e}}_2$ are zero-mean random variables. Such a relationship is obtained when the stochastic individual yield (price) is projected orthogonally onto the stochastic yield (price) index.[1] In addition, we assume that $\tilde{\boldsymbol{e}}_i$ is independent with $(\tilde{p}, \tilde{y})$ for $i = 1,2$, and $\tilde{\boldsymbol{e}}_1$ is independent with $\tilde{\boldsymbol{e}}_2$. It should be noticed that there are no specific assumptions about the stochastic dependence between the price index and the yield index. An optimal insurance policy $[I(.,.), P]$ is solution of the following maximization program:

(7) $\underset{I(.,.),P}{Max}\, Eu[(\boldsymbol{a}_1 + \boldsymbol{b}_1 \tilde{p} + \tilde{\boldsymbol{e}}_1)(\boldsymbol{a}_2 + \boldsymbol{b}_2 \tilde{y} + \tilde{\boldsymbol{e}}_2) + I(\tilde{p}, \tilde{y}) - P]$

subject to

(8) $I(p, y) \geq 0$ for all (p, y)

(9) $P = c[EI(\tilde{p}, \tilde{y})]$

where the administrative cost function $c(.)$ is defined as in equation (3). It seems natural to assume that the regression coefficients $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are positive: the individual price and the price index are positively correlated, and the individual yield is positively correlated with the yield index. Solving this maximization problem leads to the following proposition.

[1] We thus have $\boldsymbol{b}_1 = \text{cov}(\tilde{p}_i, \tilde{p}) / \text{var}(\tilde{p})$, $\boldsymbol{a}_1 = E\tilde{p}_i - \boldsymbol{b}_1 E\tilde{p}$, $\boldsymbol{b}_2 = \text{cov}(\tilde{y}_i, \tilde{y}) / \text{var}(\tilde{y})$ and $\boldsymbol{a}_2 = E\tilde{y}_i - \boldsymbol{b}_1 E\tilde{y}$.

PROPOSITION 2. *The insurance contract, solution to program (7) subject to constraints (8) and (9) when* P *is fixed,* $\boldsymbol{b}_1 > 0$ *and* $\boldsymbol{b}_2 > 0$, *takes the following form:*

A decreasing trigger function $\hat{p}(.)$ *exists such that:*

$$I^*(p, y)\begin{cases} > 0 & \text{if } \; p < \hat{p}(y) \\ = 0 & \text{otherwise.} \end{cases}$$

For all $(p, y) : I(p, y) > 0$, *the marginal indemnity functions satisfy:*

$$\text{(10)} \qquad \frac{\partial I^*(p, y)}{\partial p} = -\boldsymbol{b}_1 \frac{E_2\left[(\boldsymbol{a}_2 + \boldsymbol{b}_2 y + \tilde{\boldsymbol{e}}_2) E_1 u''(\tilde{\boldsymbol{p}})\right]}{E_1 E_2 u''(\tilde{\boldsymbol{p}})}$$

$$\text{(11)} \qquad \frac{\partial I^*(p, y)}{\partial y} = -\boldsymbol{b}_2 \frac{E_1\left[(\boldsymbol{a}_1 + \boldsymbol{b}_1 p + \tilde{\boldsymbol{e}}_1) E_2 u''(\tilde{\boldsymbol{p}})\right]}{E_1 E_2 u''(\tilde{\boldsymbol{p}})}$$

where $\boldsymbol{p} = (\boldsymbol{a}_1 + \boldsymbol{b}_1 p + \boldsymbol{e}_1)(\boldsymbol{a}_2 + \boldsymbol{b}_2 y + \boldsymbol{e}_2) + I(p, y) - P$ and E_i is the expectation operator with respect to $\tilde{\boldsymbol{e}}_i$, for $i = 1,2$.

One can also show that under actuarially fair insurance, i.e. $c'(I) = 1$ for all $I \geq 0$, the optimal revenue insurance contract provides full insurance against $\tilde{p}$ and $\tilde{y}$. The optimal coverage is partial if the insurance is costly, i.e. $c'(I) > 1$ for some $I \geq 0$.

Equations (10) and (11) can be rewritten as:

$$\text{(12)} \qquad \frac{\partial I^*(p, y)}{\partial p} = \boldsymbol{b}_1 \left\{ -E[\tilde{y}_i / \tilde{y} = y] + \text{cov}_2\left[\tilde{\boldsymbol{e}}_2, \frac{E_1 u''(\tilde{\boldsymbol{p}})}{-E_1 E_2 u''(\tilde{\boldsymbol{p}})}\right]\right\}$$

$$\text{(13)} \qquad \frac{\partial I^*(p, y)}{\partial y} = \boldsymbol{b}_2 \left\{ -E[\tilde{p}_i / \tilde{p} = p] + \text{cov}_1\left[\tilde{\boldsymbol{e}}_1, \frac{E_2 u''(\tilde{\boldsymbol{p}})}{-E_1 E_2 u''(\tilde{\boldsymbol{p}})}\right]\right\}$$

where cov_j is the covariance operator with respect to the $\tilde{\boldsymbol{e}}_j$ basis risk, for $j = 1,2$. The optimal marginal indemnity function with respect to the price index, expressed in equation (12), is affected by the regression coefficient between the individual price and the price index, by the bias between the individual yield and the yield index through the expectation of the individual yield conditional on the yield index, and by the $\tilde{\boldsymbol{e}}_2$ yield basis risk through the covariance term. This slope is higher or lower than $-\boldsymbol{b}_1 E[\tilde{y}_i / \tilde{y} = y]$ depending on whether the covariance term is positive or negative. Since the profit function increases with $\tilde{\boldsymbol{e}}_2$, this covariance is positive, null or negative as the marginal utility function, is convex, linear or concave. The notion of prudence, which is linked to the convexity of u', is recognized as a realistic behavioral assumption (Kimball). It is a necessary condition for decreasing absolute risk aversion. The role of prudence in the design of an optimal insurance contract has recently been emphasized by Mahul (2000) in the presence of an insurable risk and an uninsurable and independent risk. Therefore, the marginal indemnity function with respect to the aggregate price is higher than $-\boldsymbol{b}_1 E[\tilde{y}_i / \tilde{y} = y]$ if the producer is prudent. From a similar analysis, the marginal indemnity function with respect to the aggregate yield in (13) turns out to be higher than $-\boldsymbol{b}_2 E[\tilde{p}_i / \tilde{p} = p]$ if the producer exhibits prudence.

As a first approximation, the covariance terms in equations (12) and (13) can be neglected.[2] The partial derivatives become:

(14) $$\frac{\partial I^*(p,y)}{\partial p} \approx -\boldsymbol{b}_1 E[\tilde{y}_i / \tilde{y} = y]$$

(15) $$\frac{\partial I^*(p,y)}{\partial y} \approx -\boldsymbol{b}_2 E[\tilde{p}_i / \tilde{p} = p].$$

The closed-form solution of the optimal insurance contract thus takes the following form:

(16) $$I^{A1}(p,y) = \max\left[\hat{S} - E(\tilde{y}_i / \tilde{y} = y)E(\tilde{p}_i / \tilde{p} = p), 0\right].$$

The approximate indemnity function is contingent on the product of the conditional expectation of the individual yield and of the individual price $S(p,y) = E(\tilde{y}_i / \tilde{y} = y)E(\tilde{p}_i / \tilde{p} = p)$. Payoffs are made whenever $S(p,y)$ is lower than the trigger level $\hat{S}$. Using equations (5) and (6), this can be rewritten as

(17) $$I^{A1}(p,y) = \max\left[\hat{T} - (\boldsymbol{b}_1 \boldsymbol{a}_2 p + \boldsymbol{b}_2 \boldsymbol{a}_1 y + \boldsymbol{b}_1 \boldsymbol{b}_2 py), 0\right]$$

where $\hat{T} = \hat{S} - \boldsymbol{a}_1 \boldsymbol{a}_2$. The approximate indemnity schedule depends not only on the gross revenue index py but also on the price index and on the yield index taken separately. It is worthwhile to note that, contrary to the optimal insurance policy, the closed-form solution does not depend on the producer's attitude towards risk. This approximate form is also a consequence of Proposition 1 where the gross revenue is equal to the function S.

[2] The covariance term in (12) (in (13)) equals zero if the producer's utility function is quadratic, an unrealistic assumption, and/or if the individual price (the individual yield) is a deterministic linear function of the price index (the yield index), i.e. there is no price basis risk (no yield basis risk).

Two particular cases are derived from this result. First, if the insurance indemnity is based on the price index and on the individual yield, i.e. $\tilde{y} \equiv \tilde{y}_i$, then the closed-form solution (16) can be rewritten as

$$(18) \qquad I^{A2}(p, y_i) = \max\left[\hat{S} - y_i E(\tilde{p}_i / \tilde{p} = p), 0\right] = \max\left[\hat{S} - (\boldsymbol{a}_1 y_i + \boldsymbol{b}_1 p y_i), 0\right].$$

The approximate indemnity function is contingent on the insurable gross revenue py_i and on the individual yield. Second, if the insurance indemnity is based on the individual price, i.e. $\tilde{p} \equiv \tilde{p}_i$, and on the yield index then the closed-form solution of the optimal insurance contract becomes

$$(19) \qquad I^{A3}(p_i, y) = \max\left[\hat{S} - E(\tilde{y}_i / \tilde{y} = y) p_i, 0\right] = \max\left[\hat{S} - (\boldsymbol{a}_2 p_i + \boldsymbol{b}_2 p_i y), 0\right].$$

The approximate indemnity schedule is contingent on the insurable gross revenue $p_i y$ and on the individual price.

Optimal Insurance and Hedging Decisions

One of the first insurance product proposed to the U.S. farmers was the multiple peril crop insurance (MPCI) contract. Under this government-subsidized program, producers receive indemnities when the realized individual yield falls below a yield guarantee $\hat{y}_i$:

$$(20) \qquad I^{MPCI}(y_i) = p_s \max\left[\hat{y}_i - y_i, 0\right]$$

where p_s is the price selection at which the insurer compensates the farmer for a unit loss of the commodity. The farmer selects his yield guarantee $\hat{y}_i$ between 50% and 75% of the individual average historical yield m_i, refereed to as the actual production history (APH), in 5% increments, and any price selection between 30% and 100% of the FCIC estimated market price (GAO, Harwood et al.).

Since 1996, three government-subsidized revenue insurance plans are offered to the U.S. farmers. For both Income Protection (IP) and Revenue Assurance (RA), the farmer's gross revenue guarantee is defined when crops are planted. It equals the product of the yield guarantee $\hat{y}_i$, which is between 50% and 75% of the farmer's APH, and the projected price. The realized gross revenue is established by multiplying the realized yield and a price at harvest. The producer receives an indemnity when the realized gross revenue falls below the revenue guarantee:

(21) $I^{j}\left(p, y_i\right) = \boldsymbol{q}_i \max\left[\boldsymbol{d}^{j}\hat{p}\hat{y}_i - \boldsymbol{d}^{j} p y_i, 0\right]$ for $j = RA, IP$

where p is the Chicago Board of Trade's November price for the December contract, $\hat{p}$ is the Chicago Board of Trade's February price for the December contract, $\boldsymbol{q}_i \in \{0,1\}$ is the coverage level chosen by the producer, and $\boldsymbol{d}^{j}$ is a fixed coefficient of adjustment. This coefficient equals one under IP and it is equal to a county factor under RA. Therefore, indemnity under IP is based on individual yield and national price, whereas indemnity under RA is based on individual yield and local price. Under the third revenue insurance plan, called Crop Revenue Coverage (CRC), indemnities are triggered if the farmer's realized gross revenue falls below a revenue guarantee measured by the product of the realized individual yield and the higher of the price at planting and the price at harvest:

(22) $I^{CRC}\left(p, y_i\right) = \boldsymbol{q}_i \max\left[\max\left(p, \hat{p}\right)\hat{y}_i - p y_i, 0\right].$

where the yield guarantee $\hat{y}_i$ is selected between 50% and 85%, in 5% increments, of the farmer's APH, and $\boldsymbol{q}_i \in \{0, 0.95, 1\}$ is the coverage level chosen by the producer. This indemnity schedule can be rewritten as

(23) $I^{CRC}\left(p, y_i\right) = \boldsymbol{q}_i \max\left[\left(\hat{p} + \max\left(p - \hat{p}, 0\right)\right)\hat{y}_i - p y_i, 0\right].$

This revenue insurance thus provides a replacement-cost protection to producer that is characterized by the call option $\max\left(p - \hat{p}, 0\right)$.[3] The producer's revenue guarantee may increase over the season, allowing the producer to purchase "replacement" bushels if yields are low and prices increase during the season. An innovative insurance product has been proposed to English producers in 1999. Its indemnity schedule looks like the CRC program, except that it is contingent on an area yield index rather than the individual yield. Other revenue insurance contracts are under study and they should be launched in Europe in the near future.

U.S. revenue insurance programs continue to expand, with a new product introduced in 1999 and called Group Risk Income Protection (GRIP). Under this program, coverage is based on county-level gross revenue, calculated as the product of the county yield and the harvest-time futures market price. The GRIP program is available as a pilot program in selected counties for corn and soybean (Dismukes). Formally, its indemnity schedule is

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[3] The price at harvest is subject to a maximum upward price movement: $p \leq \hat{p} + m$ where m is equal to $1.50 per bushel for corn and $3.00 for soybeans.

(24) $I^{GRIP}(p,y)=\theta_i \max[\hat{p}\hat{y}-py,0]$

where y is the realized area yield, and $\hat{y}$ is the area yield guarantee selected between 70% and 90%, in 5% increments, of the expected area yield, and θ_i is the coverage level. This insurance policy adds a revenue component to the area yield insurance program.

Farmers have also the opportunity to hedge against yield and price variations on financial markets. Beside futures price contracts and options on futures to manage price risk, they can use innovative instruments to hedge against crop yield risk, the so-called Crop Yield Insurance (CYI) futures and options, launched by the Chicago Board of Trade in 1995. The underlying instruments are the official state-based yield estimates released during the growing and harvesting season by the U.S. Department of Agriculture (Vukina, Li and Holthausen).

Our purpose here is to examine how these revenue insurance contracts can be combined with the crop insurance policy and the hedging instruments in order to replicate the optimal hedging strategy against joint yield and price risk. We assume that the crop and revenue insurance programs do not exhibit constraints on yield and price guarantees and on the coverage level, and that insurance and hedging instruments are provided at a fair price. These assumptions will be discussed hereafter.

When the indemnity schedule of the revenue insurance contract is based on individual yield and individual price is available, we know from Proposition 1 that the first-best optimal insurance policy depends only on the individual gross revenue and this policy displays full insurance under a critical level. Consequently, crop insurance and hedging tools turn out to be redundant. Nevertheless, to our knowledge, such revenue insurance policy is not provided by real-world insurance markets.

When the indemnity schedule of the revenue insurance policy based on individual yield and aggregate price is available at a fair price, we deduce from Proposition 3 and equation (18) that the closed-form solution of the optimal indemnity function net of its premium becomes:

(25) $J^{A2}(p,y_i)=I^{A2}(p,y_i)-EI^{A2}(\tilde{p},\tilde{y}_i)=[\alpha_1 E\tilde{y}_i+\beta_1 E(\tilde{p}\tilde{y}_i)]-[\alpha_1 y_i+\beta_1 p y_i]$.

It can be rewritten as

(26) $J^{A2}(p,y_i)=\alpha_1[E\tilde{y}_i-y_i]+\beta_1[E(\tilde{p}\tilde{y}_i)-py_i]$.

The approximate net indemnity function can be replicated by purchasing the IP policy at a coverage level $\theta_i=\beta_1$, or the RA policy at $\theta_i=\beta_i/\delta^{RA}$, with a revenue guarantee

$\hat{p}\hat{y}_i = p_{\max} y_{i\max}$, where $p_{\max}$ and $y_{i\max}$ are the maximum national price and the maximum individual yield respectively. It provides full coverage against the random variable $\boldsymbol{b}_1 \tilde{p}\tilde{y}_i$. The farmer also purchase the MPCI contract with $p_s = \boldsymbol{a}_1$, if positive, and $\hat{y}_i = y_{i\max}$. It provides full coverage against individual yield variations in volume. The producer is thus fully covered against price risk through the revenue insurance contract. Nevertheless, he is only partially insured against the yield risk and therefore he is induced to purchase the MPCI contract, if $\boldsymbol{a}_1$ is positive. Revenue insurance and individual crop insurance turn out to be complementary, whereas futures price contracts and options on futures are redundant. The final wealth of the insured producer becomes

(27) $$w^{ins} = p_i y_i + J^{A2}(p, y_i) = \boldsymbol{a}_1 E\tilde{y}_i + \boldsymbol{b}_1 E(\tilde{p}\tilde{y}_i) + \boldsymbol{e}_1 y_i .$$

The only source of uncertainty borne by the insurance producer stems from the zero-mean random variable $\tilde{\boldsymbol{e}}_1 \tilde{y}_i$.

When the indemnity schedule of the revenue insurance contract depending on the yield index and the price index is available at a fair price, Proposition 3 and equation (17) yield that the approximate indemnity function net of its premium is:

(28) $$J^{A1}(p,y) = I^{A1}(p,y) - EI^{A1}(\tilde{p},\tilde{y}) = [\boldsymbol{b}_1\boldsymbol{a}_2 E\tilde{p} + \boldsymbol{b}_2\boldsymbol{a}_1 E\tilde{y} + \boldsymbol{b}_1\boldsymbol{b}_2 E(\tilde{p}\tilde{y})] - [\boldsymbol{b}_1\boldsymbol{a}_2 p + \boldsymbol{b}_2\boldsymbol{a}_1 y + \boldsymbol{b}_1\boldsymbol{b}_2 py]$$

which can be rewritten as

(29) $$J^{A1}(p,y) = \boldsymbol{b}_1\boldsymbol{a}_2[E\tilde{p} - p] + \boldsymbol{b}_2\boldsymbol{a}_1[E\tilde{y} - y] + \boldsymbol{b}_1\boldsymbol{b}_2[E(\tilde{p}\tilde{y}) - py].$$

This closed-from solution can be replicated as follows: the producer purchases the GRIP policy at a coverage level $\boldsymbol{q}_i = \boldsymbol{b}_1\boldsymbol{b}_2$ and at a revenue guarantee $\hat{p}\hat{y} = p_{\max} y_{\max}$, where $y_{\max}$ is the maximum yield index. It thus provides full insurance against $\boldsymbol{b}_1\boldsymbol{b}_2 \tilde{p}\tilde{y}$. In addition, he selects on financial markets a short (long) CYI futures position against the $\tilde{y}$ yield risk with a futures yield at planting equal to the expected aggregate yield and an optimal hedge ratio equal to $|\boldsymbol{b}_2\boldsymbol{a}_1|$ if $\boldsymbol{a}_1$ is positive (negative), and a short (long) futures positions against the $\tilde{p}$ price risk with a futures price at planting equal to the expected aggregate yield and an optimal hedge ratio equal to $|\boldsymbol{b}_1\boldsymbol{a}_2|$

if α_2 is positive (negative).[4] Therefore, revenue insurance, crop yield insurance futures contracts and price futures contracts are complementary. The final wealth of the insured producer is

$$w^{ins} = p_i y_i - J^{A1}(p,y) = \alpha_1\alpha_2 + \beta_1\alpha_2 E\tilde{p} + \beta_2\alpha_1 E\tilde{y} + \beta_1\beta_2 E(\tilde{p}\tilde{y}) + p_i\varepsilon_2 + E[\tilde{y}_i / \tilde{y} = y]\varepsilon_1 . \tag{30}$$

The insured producer bears two sources of risk caused by the uninsurable and unhedgeable price and yield basis risks.

When the revenue insurance program is based on individual yield and national price, the insurance and hedging strategy is replicated with the IP or RA contract rather than the CRC policy. However, this insurance product turns out to have the highest enrollment among the U.S. farmers, accounting for about one-third of the total crop insurance sales in the areas where they are offered (GAO). This observation seems *a priori* to be in contradiction with the above theoretical results. This may be the consequence of existing constraints on yield guarantee and coverage level in the actual IP and CRC programs. Since these insurance policies are sold at a price which is less than the actuarial premium thanks to the premium subsidies, farmers will be induced to select the policy which provides us the larger coverage. By construction, the CRC product generates a higher coverage than the IP and RA products. Therefore, we can conjecture that, for an identical level of coverage, the IP or RA product should be preferred to the CRC product. This seems to be confirmed by recent empirical results obtained by Heifner and Coble.

These authors also show that revenue insurance based on individual yield does not substitute completely for forward pricing. Our previous theoretical results seem to be in contradiction with their empirical results. Once again, this may due to the existence of upper bound on yield guarantee. This seems to be confirmed by their simulations about the effect of yield guarantee on the optimal hedge ratio (see Heifner and Coble; Figures 19 to 22). When a revenue insurance contract like the IP program is available, the hedge ratio tends to zero as the yield guarantee is higher than 125% of the historic individual yield in most of the counties.[5]

Under the IP and RA insurance programs, the coverage level is equal to one, i.e. the producer insures all his crop, or zero, i.e. the producer does not purchase the insurance policy. This constraint may induce him to prefer the RA or IP plan depending on whether β_1 is close to

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[4] The CYI hedging ratio is defined up to a multiplicative factor which depends on the unit of trading of the financial contracts. For example, the unit of trading for Iowa Corn Yield Insurance Futures is the Iowa yield estimate times $100.

[5] These figures also show that, under the CRC program, the optimal hedge ratio increases with the yield guarantee.

coefficient of adjustment or to one. He may refuse to purchase both of them if b_1 is close to zero or the GRIP plan if $b_1 b_2$ is close to zero.

Conclusion

This paper has examined the design of an optimal revenue insurance contract when the producer faces joint yield and price risk. This is a first step in a long-term research project on the rational insurance purchasing decisions against multiple risks in the context of incomplete markets. The message of this paper is twofold. First, if the indemnity schedule is contingent on individual yield and individual price, the optimal insurance contract has been shown to depend only on the individual gross revenue. It displays full insurance under a trigger revenue. In this context of complete markets, crop yield insurance contracts and hedging instruments against price risk turn out to be redundant. Second, we have proved that this result fails when the indemnity schedule is contingent on yield index and/or price index. In this context of incomplete markets, the optimal revenue insurance contract does not depend only on the gross revenue index. A closed-form solution of the optimal insurance contract against yield and price indexes has been derived. It can be replicated with unconstrained yield and revenue insurance policies and hedging instruments sold at a fair price. The IP and RA programs have turned out to be complementary with the MPCI program and price hedging contract are redundant. The GRIP plan has been shown to be complementary with yield and price hedging tools.

If constraints on the yield guarantee and on the coverage level exist or if the insurance and hedging contracts are sold at a price higher than the fair one, then the closed-form solution of the optimal revenue insurance policy cannot be replicated any longer. These restrictions on the insurance contracts and hedging instruments create a new source of incompleteness that will be investigated in further research.

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