Management of Multiple Price Risks for Firms:

An Improved Method for Estimating Hedge Ratios

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I. Introduction

All agribusiness firms face multiple price risks, which, depending on the firms' position in the market channel, can include input prices, output prices, interest rate, and exchange rates. Using the futures market to reduce profit and price volatility can be a least cost alternative among several. Effective *ex ante* hedge ratios can be generated for optimal positions in the futures market, leading to successful management of the firm's single and multiple-commodity price risks. This paper compares traditional estimation procedures based on parametric techniques to an improved technique based on a nonparametric procedure, which uses fewer restrictive assumptions.

The hedge ratio is traditionally estimated by simple regression such as OLS and SUR. However, this procedure is statistically inefficient because it ignores heteroskedasticity, which implies that estimates could not explain information inflow in prices. Since futures and spot prices exhibit time varying volatility, they should be represented by conditional heteroskedastic shocks (the conditional covariance of futures and cash prices), leading to optimal hedge ratios that may change over time. With the development of autoregressive conditional heteroskedasticity (ARCH) and generalized ARCH (GARCH) models, various studies have been conducted to get efficient estimates of time varying hedge ratios (McNew and Fackler, 1994; Garcia, Roh, and Leuthold, 1995; Bera, Garcia, and Roh, 1997).

Presently, these analyses have been performed in a parametric context, involving a complete specification of the process of interaction among important variables, which may lead to error in the hedging decision. Specifically, these analyses are based on several assumptions. The functional form of the conditional mean of cash prices given futures prices is assumed to be known, usually linear. The conditional variance of cash prices given futures prices and the autocorrelation of error terms are assumed specified. The parametric joint density (data generating process) of cash prices given futures prices is assumed normal. Last, prices are often considered to be nonstochastic even though ARCH-type models incorporate stochastic features. Due to these assumptions, optimal hedge estimation in a parametric framework may not be robust to slight inconsistencies between data and any particular parametric specification. Thus, any misspecification in this regard may lead to erroneous estimation of the hedge ratio, usually exaggerating the variance.

This study develops and tests a new method for estimating time varying hedge ratios, a kernel-based nonparametric partial derivative estimator, specifically locally polynomial kernel regression, which reduces the number of arbitrary parametric restrictions. This estimator is consistent under most circumstances, such as rigid dependence of the error terms and an unknown functional form, and gives a better fit to the time series that often have fat tails. Thus, hedge ratios that are dependent on past values of conditioning variables can be consistently estimated within this nonparametric framework. The potential benefits of using this estimator are that it avoids the potential errors introduced by functional misspecification and expands the settings in which the estimator can be shown to be consistent, thus, permitting more efficient risk management (Ullah, 1988; Wand and Jones, 1995). This technique has never before been applied to optimal hedge ratio estimation and can be applied to both single commodity and simultaneous production decisions on outputs and inputs. As an example, it is applied here to the hog feeding complex.

If the functional form of the conditional mean of cash prices given futures prices is known, then the parametric approach will perform better or at least as well as the nonparametric approach. Therefore, a simple linear model and a GARCH-type model are developed to estimate time varying hedge ratios to compare with the nonparametric estimation.

The accuracy of hedge ratios, estimated by these three different methods, are evaluated in terms of maximizing expected return and variance reduction comparison. In addition, the accuracy of hedge ratios are evaluated and compared to various hedging strategies including naïve hedging, singl-commodity hedging, and no hedging.

II. Hedging Rules

Hog producers face multiple price risks due to the volatile prices of live hogs and feed grains, and often achieve the objective of reducing these price risks by forward pricing through either the futures market or forward cash market[1]. Since buyers of hogs such as meat packers charge for their services, prices offered through forward cash contracts may be less than those offered using futures market, and hence the futures market is often preferred to the forward cash market. Another advantage of using the futures market comes from marketing flexibility.

In this study the feeding (final) stage of hog production (wean-to-finish) is considered because it is the main stage of hog production where large amounts of feed grains are consumed. It takes around 4 months to reach final market weight of hogs of about 225 pounds, a stage which begins when the hogs weigh about 60 pounds. Among various feed ingredients, corn is the major feed grain, and around 615 pounds per hog are fed during this final period[2]. Corn provides dietary energy in the form of carbohydrates and fat. The hedging decision framework is

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[1] Forward pricing is not the only alternative to managing pricing risk. Floor pricing through the options market provides a minimum price while allowing the producer to take advantage of any higher prices. Forward pricing on the other hand will provide more price protection against lower prices than will floor pricing, but also precludes gains from higher prices.

[2] Another potentially important input is soybean meal. However, the amount of soybean meal consumed per hog is approximately only 10% of total feed grains while corn takes around 85%. Also, in a similar analysis for live cattle, Noussinov and Leuthold (1999) found that the coefficient for the soybean meal hedge ratio was insignificant and did not affect the overall hedging results. In addition, soybean meal adds a third dimension to the kernel estimation, which would make the procedure used in this study very complex.

composed by two stages. The first stage, from t-6 to t-4, constitutes a planning period before feeding begins (t refers to when the output is marketed, and time is measured in months). At t-6, hedging occurs by simultaneously taking long positions in the input and a short position in the output in the futures market. Hedges on inputs are held for two months until the feeding begins. Corn is purchased for the feeding of hogs at t-4 in the cash market. At the same time, those input hedges taken at t-6 are liquidated. After feeding, the live hogs are sold in the cash market at t and the associated output futures position held for six months is lifted.

Then, the returns from cash and futures transactions of hogs and corn at time t can be written as

$$R_t = P_{H,t} - P_{C,t-4} + \boldsymbol{b}_{H,t-6}(F_{H,t-6} - F_{H,t}) + \boldsymbol{b}_{C,t-6}(F_{C,t-4} - F_{C,t-6}), \qquad (1)$$

where P and F stand for cash and futures prices, respectively, and H and C are abbreviated for hogs and corn respectively. $\boldsymbol{b}_{H,t-6}$ and $\boldsymbol{b}_{C,t-6}$ are hedge ratios for hogs and corn at time t-6. This equation assumes no transaction costs. Assuming unbiasness in futures market, hedge ratios for multiproduct can be generated by the mean-variance framework. The variance of returns in (1) can be written as

$$\begin{aligned} Var(R_t) = {} & \operatorname{var}(P_{H,t}) + \operatorname{var}(P_{C,t-4}) + \boldsymbol{b}_H^2 \operatorname{var}(F_{H,t}) + \boldsymbol{b}_C^2 \operatorname{var}(F_{C,t-4}) - 2\operatorname{cov}(P_{H,t}, P_{C,t-4}) - 2\boldsymbol{b}_H \operatorname{cov}(P_{H,t}, F_{H,t}) \\ & + 2\boldsymbol{b}_C \operatorname{cov}(P_{H,t}, F_{C,t-4}) + 2\boldsymbol{b}_H \operatorname{cov}(P_{C,t-4}, F_{H,t}) - 2\boldsymbol{b}_C \operatorname{cov}(P_{C,t-4}, F_{C,t-4}) + 2\boldsymbol{b}_H \boldsymbol{b}_C \operatorname{cov}(F_{H,t}, F_{C,t-4}). \end{aligned} \qquad (2)$$

Solving the first-order conditions of (2) for the optimal hedge ratios yields the following decision rules for hog producers:

$$\boldsymbol{b}_C^* = \frac{[\operatorname{cov}(P_{H,t}, F_{H,t}) - \operatorname{cov}(P_{C,t-4}, F_{H,t})]\operatorname{cov}(F_{H,t}, F_{C,t-4}) + [\operatorname{cov}(P_{C,t-4}, F_{C,t-4}) - \operatorname{cov}(P_{H,t}, F_{C,t-4})]\operatorname{var}(F_{H,t})}{\operatorname{var}(F_{C,t-4})\operatorname{var}(F_{H,t}) - \operatorname{cov}(F_{H,t}, F_{C,t-4})^2}, \qquad (3)$$

and

$$\boldsymbol{b}_H^* = \frac{\text{cov}(P_{H,t}, F_{H,t}) - \text{cov}(P_{C,t-4}, F_{H,t}) - \boldsymbol{b}_C \ \text{cov}(F_{H,t}, F_{C,t-4})}{\text{var}(F_{H,t})}. \tag{4}$$

Hedge ratios of single commodity is simply $\text{cov}(P, F)/\text{var}(F)$, where time subscripts for hedge ratios are omitted for simplicity.

III. Econometric Model and Data

Locally Polynomial Kernel Estimation

Consider *n* data points $\{(X_i, Y_i)\}_{i=1}^{n}$ and a general regression model,

$$Y_i = m(X_i) + u_i \tag{5}$$

where $m(x) = E(Y|X = x)$, $E(u|x) = 0$ and $X = \{F_t, Z_{t-l}\}$. The stochastic nature of the relationship is represented by the zero-mean random shock u_t. Y is easily replaced by P_t, and X is replaced by F_t and Z_{t-l}, where Z_{t-l} is other relevant information, and $l = 1, \cdots, m$. The conditional expectation of the partial derivative of P_t w.r.t. F_t can be derived to obtain hedge ratios,

$$HR_t = \frac{\partial E(P_t | F_t, Z_{t-1}, \ldots, Z_{t-m})}{\partial F_t}(F_t, Z_{t-l}) = \frac{\partial m}{\partial F_t}(F_t, Z_{t-l}). \tag{6}$$

The value of the function HR_t, which is dependent on F_t and Z_{t-l} at a particular point, gives the value of the partial derivative of the conditional expectation functional with respect to the concurrent futures price variable.

The aim of a regression analysis is to produce a reasonable approximation to the unknown response function $m(X_i)$. In the parametric approach, the critical assumptions of the response function $m(X_i)$ are known functional form and normality of error terms. Any misspecification in $m(X_i)$ causes serious consequences for econometric inference; for example, the estimators of the regression parameters can be seriously biased. Since the nonparametric approach does not rely on the assumptions underlying the parametric approach, it has more flexibility and is more efficient in estimating the complicated unknown response function $m(X_i)$.

Locally polynomial kernel estimation can produce a good fit to a sample characterized by nonlinear relationships. Using a local polynomial kernel, the locally optimal hedge ratio can be achieved and represented by partial derivatives or derivatives in given directions. This would thereby provide all the relevant information whether or not a single globally optimal concept is applicable, so that a local polynomial kernel is used to test if hedge ratio is constant over an entire sample.

Local polynomial kernel estimates the regression function at a particular point by "locally" fitting a p^{th} degree polynomial to the data via weighted least squares.

To demonstrate the model, the model (5) is modified as follows,

$$Y_i = m(X_i) + s^{1/2}(X_i)u_i, \; i = 1, \ldots, n \tag{7}$$

where, $m(x) = E(Y|X = x)$, $s(x) = Var(Y|X = x)$, and $\{u_i\}$ are i.i.d. The objective of this model is to estimate partial derivatives up to the p^{th} order $d^p m(x)/dX_i^p$ without imposition of $m(x)$ and $d^p m(x)/dX_i^p$ belonging to the parametric family of functions.

Explicit formulae to estimate hedge ratios in (6) for local linear (p=1) is[3]:

$$HR_t = \hat{b}(x;H) = n^{-1}\sum_{i=1}^{n} \frac{[\hat{s}_0(x;H)(x - X_i) - \hat{s}_1(x;H)]K_H(x - X_i)Y_i}{\hat{s}_2(x;H)\hat{s}_0(x;H) - \hat{s}_1(x;H)^2}, \tag{8}$$

where $\hat{s}_j = n^{-1}\sum_{i=1}^{n}(x - X_i)^j K_H(x - X_i)$. H is a $d\times d$ symmetric positive definite matrix depending on n, which is a set of univariate h. K is a d-variate kernel and

[3] Because of space limitation, details on the proof and explanation of (8) is available from the author.

$K_H(v) = \left|H\right|^{-1/2} K(H^{-1/2}v)$. The kernel, *K*, is a continuous, bounded, and symmetric probability density function. The assumptions are as follows

$$\int K(v)dv = 1, \int vK(v)dv = 0, \text{ and } \int v^2 K(v)dv = k_2 \ .$$

Kernel estimator is described as a sum of ‘bumps’ placed at the observations, and the kernel function *K* determines the shape of the bumps, the shape of the weights, while the window width *h* determines their width, the size of the weights. As *h* becomes large, the smoothness of estimation will increase.

GARCH Model

In this study, the *BEKK* technique, is employed in BGARCH and MGARCH specification[4]. Time series diagnostics led to the following econometric specification of the model, which has ARMA and exogenous variables in the conditional mean:

$$\begin{aligned} \Delta P_t &= C + \sum_{i=1}^{r} \boldsymbol{f}_i P_{t-i} + \sum_{i=0}^{l} \boldsymbol{b}_i F_{t-i} + \sum_{i=1}^{s} \boldsymbol{q}_i e_{t-i} + e_{tP} \\ \Delta F_t &= C + \sum_{i=1}^{r} \boldsymbol{f}_i F_{t-i} + \sum_{i=0}^{l} \boldsymbol{b}_i P_{t-i} + \sum_{i=1}^{s} \boldsymbol{q}_i e_{t-i} + e_{tF} \end{aligned} , \qquad (9)$$

where *C* is constant mean, *P* and *F* are cash and futures prices, and < denotes changes in prices. $e_t = [e_{tP}\ e_{tF}]^T \sim MN(0, V_t)$ and $V_t = AA^T + A_1(e_{t-1}e_{t-1}^T)A_1^T + B_1V_{t-1}B_1^T$. A, A_1, B_1 are $d \times d$ matrices, and V_t is symmetry and non-negative-definiteness of the conditional covariance matrix.

The log likelihood function for the *BEKK* model (1995) is:

$$L(\boldsymbol{q}) = -\frac{TN}{2}\log 2\boldsymbol{p} - \frac{1}{2}\sum_{i=1}^{T}\left(\log\left|V_t\right| + e_t^T V_t^{-1} e_t\right), \qquad (10)$$

where $\boldsymbol{q}$ denotes all unknown parameters in e_t and V_t, *T* is the sample size, and *N* is the number of mean equations. Conditional normality has been assumed.

Data Description

The hog producer is assumed to begin planning for, and subsequently feeding, a new lot of hogs every week. Percentage changes in weekly (Wednesday) cash and futures closing prices are used for January 1990 to June 1999 (last two years for lean hogs), providing 493 number of observations: 332 for live hogs and 161 for lean hogs[5]. Wednesday is selected because on that day both cash and futures trading is active with relatively high and stable trading volume. Omaha cash and central Illinois bid prices serve as the cash prices for hogs and corn, respectively, because of their relatively high volume and wide acceptance as market barometers.

Lean hog prices are converted to live hog prices by multiplying by 0.74 to get overall hog hedge ratios. Lean hog values represent the carcass, averaging a 74% yield from live hogs. Futures contracts selected are those that will be the nearby ones when hedges are lifted, and these futures positions are maintained throughout the hedging period without adjustment. Data during the delivery months are not used.

IV. Estimation Results and Nonconstant Hedge Ratios

Primary Time Series Analysis

The presence of a unit root is tested in each price level and change by performing Augemented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. These tests confirm that all data are stationary except hog futures. Hence, these data are converted into percentage changes to be stationary. Jarque-Bera normality tests on percentage changes of each series indicate that the null hypothesis of normality is rejected for corn cash, hog cash, and hog futures. The normality hypothesis is again rejected for the parametric joint density of cash given futures in the

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[4] This model is referred to as BEKK specification due to the results obtained by Baba, et al (1989).
[5] The final live hog contract is December 1996 and the first lean hog contract is February 1997.

GARCH model, implying an assumption violation. Conditional heteroskedasticity is identified under the assumption of normality, but since the assumption is broken, this result is not valid.

Model Selection

Model choice for kernel regression methods is the subject of ongoing research. Three methods are used: R-square, approximate F-test, and cross-validation. The following models are selected among several,

$$P_t = f(F_t, P_{t-i}, F_{t-j}),$$

where $i = j = 1$ for corn and $i = j = 1, \cdots, 3$ for hogs in single commodity hedging. Also $i = 1$ and $j = 1, \cdots, 2$ are selected for both corn and hogs in multiproduct hedging.

In this study two types of models are considered with various lags. One is the traditional GARCH model with constant mean, C, and the other is ARMA and exogenous variables in the conditional mean. Using AIC and BIC model selection, the second model is selected with ARMA (1,1) and lag 1 of futures prices for both single-commodity and multiproduct hedging.

$$\Delta P_t = C + \boldsymbol{f}_1 P_{t-1} + \boldsymbol{b}_1 F_{t-1} + \boldsymbol{q}_1 e_{t-1} + e_{t,P}$$
$$\Delta F_t = C + \boldsymbol{f}_1 F_{t-1} + \boldsymbol{b}_1 P_{t-1} + \boldsymbol{q}_1 e_{t-1} + e_{t,F}.$$

AIC and BIC model selection is again used to select simple linear models as follows

$$P_t = g(F_t, P_{t-i}, F_{t-j}),$$

where $i = j = 1$ for corn, and $i = 1$ and $j = 1, \cdots, 3$ for hogs in single commodity hedging. Also $i = j = 1$ is selected for both corn and hogs in multiproduct hedging.

Estimated Hedge Ratios

Figures 1 through 4 present the estimated hedge ratios of two alternatives, Local Polynomial Kernel (LPK) and GARCH, which are nonconstant and time varying hedge ratios, respectively. Both corn and hogs are clearly nonconstant for both alternatives. Figures 1 and 2 show the relationship between hedge ratios and percentage futures price changes as given by LPK. These ratios give an idea to hog producers the many different positions (hedge ratios) they might take in the futures market as futures prices change. For example, if hog producer wants to hedge the input, corn, and expects corn futures price to change by 5.2% (0.052) in two months from today, the producer needs to take a position in the corn futures market by 0.90 as indicated in figure 1. Figures 3 and 4 present the relationship between hedge ratios and estimated time period, as given by GARCH.

The estimation results for single-commodity and multiproduct hedges from OLS, LPK and GARCH during the estimation years from January 3, 1990 to June 30, 1999 are reported in table 1. The hedge ratios using LPK reported here are estimated at the mean level of each price series. That is, $P_t = f(\overline{F}_t, \overline{P}_{t-i}, \overline{F}_{t-i})$, where $i = 1,\cdots,3$. As we go from single commodity to multiproduct hedging, LPK produces bigger hedge ratios for corn (0.94 to 1.16) and smaller hedge ratios for hogs (0.79 to 0.55). In the mean time, GARCH generates smaller hedge ratios for both commodities (1.11 to 0.93 for corn and 1.05 to 0.62 for hogs) and OLS remains at the same ratios (0.95 for corn and 0.91 for hogs).

For single commodity hedging, LPK generates the smallest estimated hedge ratio (0.94 for corn and 0.79 for hogs), both being significantly less than one. Meanwhile, BGARCH produces the largest hedge ratio (1.11 for corn and 1.05 for hogs), which is significantly larger than one for corn and but not statistically different from one for hogs. Interestingly, the corn hedge ratios for all three alternatives are significantly different from one while only the hog hedge ratio generated by LPK is significantly different from one. Thus, if only hogs are hedged based on a simple regression and BGARCH, the returns defined in (1) are not statistically different from a naïve hedge.

Unlike the single commodity hedging, MGARCH in the multiproduct case yields the smallest hedge ratios for both corn. Corn is statistically indistinguishable from the naïve hedge, while hogs are significantly less than one. Hog producers should over hedge corn and under hedge hogs at the same time when LPK is used. The estimated hedge ratios from LPK and MGARCH for hogs are far less than the one, different from OLS in the multiproduct hedging.

Hedging Effectiveness

The hedging effectiveness of the various models is examined based on two measures: the proportional reduction in the unhedged variance of returns and the proportional increase in the unhedged return. The larger the reduction in variance and the larger the increase in return, the higher the degree of hedging effectiveness. Equation (1), with hedge ratios specified by the various procedures, is used to calculate the weekly return and its variance. The variance of returns also is calculated for a naïve hedge which offsets the spot price risk by taking futures positions in corn and hogs based on the fixed proportions of the production technology.

The results of the in-sample and out-of-sample hedging are presented in tables 2 and 3. Two sample periods are used to test hedging efficiency, period of live hog trading only from January 3, 1990 to May 30, 1996, and the full sample from January 3, 1990 to June 30, 1999. During the in-sample period, based on 280 observations for live hogs in table 2 and 436 observations for all hogs in table 3, respectively, no dramatic difference is found in returns across the alternative models within either sample period. Naïve and BGARCH models show the largest increase in returns, and single commodity hedging generates more return than multiproduct hedging for both sample periods. However, the large reduction in variance is associated with multiproduct hedging relative to single commodity hedging. MGARCH performs slightly better than MLPK for both measures, return and variance, but is hard to conclude that MGARCH is superior to MLPK. Caution is needed to prefer MGARCH to MLPK because the major assumption of GARCH, normality of error terms, has been violated. Thus, using MGARCH based on the violated assumption may cause erroneous results in managing price risk. In the mean time, since LPK does not depend on any parametric assumptions, it is more useful for firms to use in multiple price risk management. Hence, both MLPK and MGARCH can identify the importance of incorporating multiple price risks in the estimation of the hedge ratios.

Interestingly, LPK for single commodity hedging performs well for both measures and for both sample periods. It outperforms BGARCH in variance reduction, and produces larger percentage increase in return than multiproduct hedging models and similar variance reduction to multiproduct hedging model. Thus, it seems that SLPK is a good alternative, which balances fairly high return and variance reduction, on the mean-variance frontier.

The out-of-sample results, which are based on 52 observations for the live hog period and 57 observations for the full period, are different. The percentage increase in return varies over alternatives, but the use of the multivariate models and SLPK result in the largest reduction in variance. MGARCH generally performs slightly better than MLPK for both measure and in both periods, but again MLPK might be better to use because of the violated assumption in the GARCH model. As with in-sample for the live hog period, SLPK presents lower return with larger decrease in variance than BGARCH, but SLPK gives better results than BGARCH for both measures in the full period, and produces similar percentage increase in return and variance reduction to MLPK and MGARCH.

Consistent with the results that Garcia, Roh, and Leuthold (1995) and Tzang and Leuthold (1990) found, the results of this analysis suggests that multiproduct hedging leads to the largest reduction in variance, indicating the importance of incorporating multiple price risk in estimation of the hedge ratios. In this study, two methods, MLPK and MGARCH, produce similar results. In addition, the in-sample findings demonstrate that SLPK performs best on the mean-variance frontier, showing good combinations of return and risk that a hog producer could assume. The out-of-sample findings confirm the in-sample results except, the SPLK model performs only slightly poorer than MLPK and MGARCH.

V. Conclusion

This study has examined the use of nonconstant optimal hedge ratios for the hog industry. A nonparametric, locally polynomial kernel, approach is used and compared to parametric approaches, BGARCH and MGARCH, and OLS models. Nonparametric models have not previously been applied to hedge ratio estimation, and used in price risk management. MGARCH and MLPK are found to be more effective than single-commodity hedging models to specify and estimate hedge ratios when dealing with multiple price risks. However, the SLPK model applied to single-commodity hedging performed nearly as well as these two multiproduct models. This study suggests taking a special care when using MGARCH as a price risk management tool because one of parametric assumptions, normality, is violated. Because of the potential assumption violations associated with the estimation and implementation of hedge ratios by GARCH models, LPK is a reasonable alternative for estimating hedge ratios to manage price risks. Certainly, using LPK for single-commodity hedging shows promise in this application.

This study suggests a new method, locally polynomial kernel, to estimate nonconstant hedge ratios, which is independent of parametric assumptions. This technique can have broad application to many types of agribusiness firms, and needs to be tested in other situations.

Further study is currently underway to see whether or not LPK performs reasonably well compared to GARCH when normality is maintained.

Table 1. Estimated Hedge Ratios for Corn and Hogs (1990 – 1999)

		Single Commodity Hedge BGARCH(1,1) for GARCH		Multiproduct Hedge MGARCH(1,1) for GARCH	
		Corn	Hog	Corn	Hog
OLS	Hedge Ratio	0.95	0.91	0.95	0.91
	t-Ratio (β = 0)	36.99	10.07	36.19	9.85
	t-Ratio (β = 1)	1.86	0.89	1.89	0.90
LPK	Hedge Ratio	0.94	0.79	1.16	0.55
	t-Ratio (β = 0)	24.83	6.31	27.79	1.96
	t-Ratio (β = 1)	1.66	1.73	3.82	1.63
GARCH	Hedge Ratio	1.11	1.05	0.93	0.62
	t-Ratio (β = 0)	30.21	14.30	7.27	6.43
	t-Ratio (β = 1)	3.07	0.69	0.57	3.94

Table 2. Hedging Effectiveness, Live Hog Period

(The unit of return is dollar)

	In-Sample (1/3/1990 ~ 5/30/1995)				Out-of-Sample (6/1/1995 ~ 5/30/1996)			
	Return	PI	Var	PR	Return	PI	Var	PR
Unhedged	66,067.77		0.0512		68,599.52		0.0755	
Naïve	66,669.02	0.91	0.0484	5.47	83,493.85	21.71	0.0651	13.77
SOLS	66,140.72	0.11	0.0495	3.32	78,033.96	13.75	0.0697	7.68
BGARCH	66,610.91	0.82	0.0487	4.88	83,384.74	21.55	0.0660	12.58
SLPK	66,586.77	0.78	0.0403	21.29	77,826.89	13.45	0.0626	17.09
MOLS	66,189.21	0.18	0.0407	20.51	78,502.29	14.44	0.0619	18.01
MGARCH	66,269.33	0.31	0.0381	25.59	77,610.20	13.14	0.0589	21.99
MLPK	66,247.12	0.27	0.0392	23.44	78,121.31	13.88	0.0600	20.53

PI, PR and Var are percentage increase, percentage reduction, and variance, respectively. S and M denote single commodity hedging and multiproduct hedging, respectively.

Table 3. Hedging Effectiveness, Full Sample Period

(The unit of return is dollar)

	In-Sample (1/3/1990 ~ 5/30/1998)				Out-of-Sample (6/1/1998 ~ 6/30/1999)			
	Return	PI	Var	PR	Return	PI	Var	PR
Unhedged	64,873.11		0.1160		36,685.04		0.1315	
Naïve	65,454.01	0.90	0.0999	13.88	38,148.31	3.84	0.1039	20.99
SOLS	65,280.61	0.63	0.0914	21.21	38,876.55	5.74	0.1186	9.81
BGARCH	65,498.90	0.96	0.1045	9.91	37,843.78	3.04	0.1010	23.19
SLPK	65,265.55	0.60	0.0898	22.59	39,015.04	6.11	0.0888	32.47
MOLS	65,091.19	0.34	0.0921	20.60	38,813.63	5.58	0.0919	30.11
MGARCH	65,169.26	0.46	0.0884	23.79	39,236.39	6.69	0.0853	35.13
MLPK	65,150.83	0.43	0.0902	22.24	39,126.54	6.40	0.0863	34.37

PI, PR and Var are percentage increase, percentage reduction, and variance, respectively. S and M denote single commodity hedging and multiproduct hedging, respectively.

Figure 1:Hedge Ratios for Corn using LPK

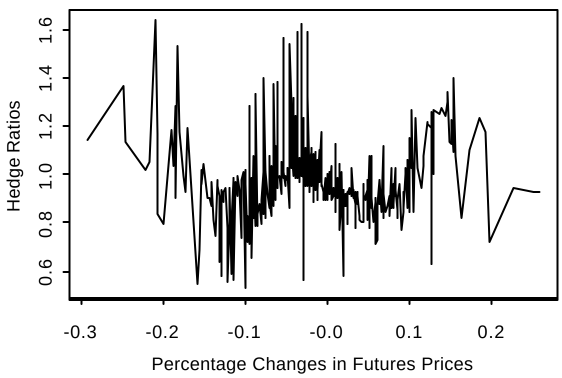


Figure 2:Hedge Ratios for Hog using LPK

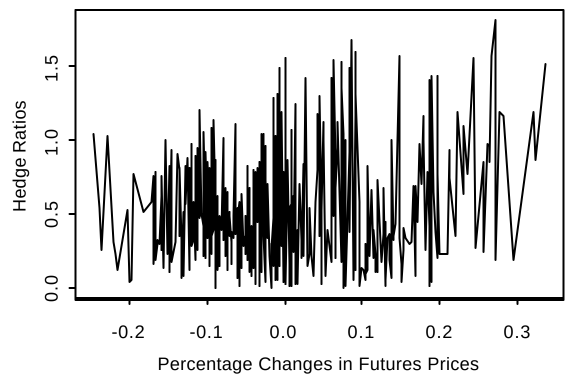


Figure 3:Hedge Ratios for Corn using GARCH

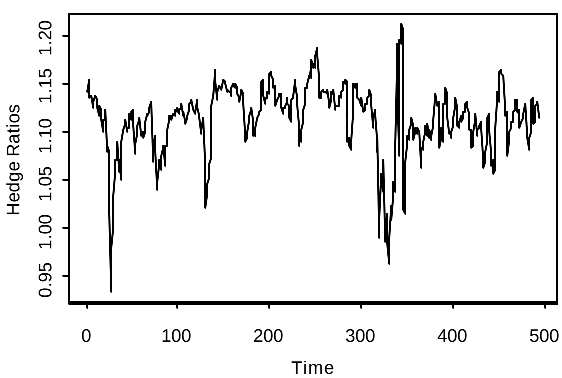
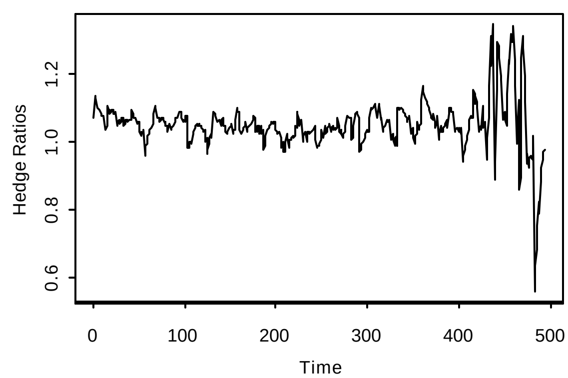


Figure 4:Hedge Ratios for Hog using GARCH



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